Porro, cum Angulum ſic, ut dictum eſt, definiverat, p. 67; ſubjungit, p. 68. *Quodſi magnitudines illæ ſint duæ lineæ, comprehenſus ab iis angulus, Planus vocabitur:* quaſi quidem de *Triangulis ſphæricis* nil unquam inaudiverit; nec alius eſſe poſſit ſuperficialis angulus, quam in *Plano.*

Adhæc, illud *duarum pluriumve*, de *Lineis* non tuto dicitur. *Trium* enim linearum concurſus, non angulum, ſed angulos ſaltem duos, conſtituunt; non enim lineæ plures duabus ad unum ſuperficialem angulum conſtituendum concurrunt. Item, cum p. 67. Angulum in genere per *duarum pluriumve, &c.* definiverat; Angulum p. 68. *una vel pluribus* ſuperficiebus comprehenſum ait (& unâ quidem angulum verticalem Coni comprehenſum;) quaſi quidem *una*, fuerit, *duæ* vel *plures.*

Inſuper, quid demum illud eſt, quod per *breviſſimam diſtantiam* inſinuatum vult? Quippe in ipſo concurſus puncto, *Nulla eſt diſtantia*; extra illud, *nulla minima*: nulla utique aſſignari poterit, qua non ſit minor: ſed revera tota hæc, quam de *Angulo* notionem concipit, eſt parum ſana. Definiendus utique eſt non per *diſtantiam* ſeu *remotionem*, ſed per *Inclinationem.* qnod ex *Euclidis* definitione didiciſſet.

Deniq;(ne multus nunc ſim) p. 171. in duabus his Quadraticarum æquationum formulis $aa - ca + dd = 0$, & $aa + ca + dd = 0$; utramque radicem *affirmativam* eſſe pronunciat. quod omnino ſecus eſt. Et quidem in priore, Radix utraque Affirmativa; ſed in poſteriore, Negativa utraque.

Atque hæc quadem, ex multis pauca, ſi non ſufficiant, ut ex ungue Leonem æſtimes, plura facile congerentur. Num autem hos *Incuria*, an *Inſcitia*, errores fuderit (prout ipſe *pag. ult.* diſtinguit) non determino. Vale.

Hæc Dn. *Walliſius* epiſtola una; cui poſtea ſubmiſit alteram, 18. *Julii* ad me ſcriptam, quam iſtoc menſe, ob alia, non licebat typis committere; nec quidem licet hoc ipſo: ne ſcil. haſce ſchedulas, publicationi variorum, idque imprimis ſermone *Anglico*, deſtinatas, diſceptationibus *Latinis* compleamus. Proxima occaſione, quæ idem *Author* porro notanda invenit vel in *unico primo* Capite *Synopſeos Laurentianæ*, Lectori (cum particularia flagitet Dn. Du Laurens) ob oculos ſiſtemus.

An Account of Two Books.

I. *R. de GRAAF Med. D. de VIRORUM ORGANIS GENERATIONI INSERVIENTIBUS, &c.* Ludg. Bat. 1668. *in* 12°.

This Treatiſe was promiſed by the Author in a printed *Epiſtle* of his, which we gave an account of in *April* laſt, *Num.* 34. p. 663. There being at the ſame time publiſht a *Prodromus* of *Joh. Van Horne*, ſuſpecting, that the Obſervations of *De Graef* were much the ſame with his upon this Subject; we do now upon the peruſal of this Book, find chiefly theſe conſiderable Differences between them.

First, the said *Van Horne* makes the *Spermatick Artery* in man to goe to the *Testicles* in a *winding*, but *De Graaf*, in a *streight* way.

Secondly, the *former* affirms, that the *vasa deferentia* have no communication with the *vesiculæ seminales*; but the latter maintains, and demonstrateth it to the Ey, there is so great a commerce betwixt them, *ut semen dum à Testibus per vasa differentia affluens in Urethram effluere nequit, propter carunculam clausam; necessariò influat in Vesiculas, in iisque pro futuro coitu reservetur.*

Thirdly, the *former* is of opinion, *triplicem esse materiam seminis*; but *De Graaf* will have but *one only*; answering the Arguments, used both by *Van Horne* and *Dr. Wharton* to prove that *triplicity*.

But that, which *De Graaf* much insists on in this Book, is, to shew what is the *true Substance* of the *Testicles*, and to vindicate the Discovery thereof to himself; affirming positively, that no man, before him ever knew the truth of it. * For the making out of which, he first denyeth, that the *Testes* are *glandulous*, or *pultaceous*; and then affirms that their substance is nothing else but a *Congeries minutissimorum vasculorum semen conficientium, quæ si absque ruptione dissoluta sibi invicem adnecteretur, facile viginti ulnarum longitudinem excederent.* Which he affirms, he can prove by ocular Demonstration.

* See the Letter of Doctor *Tim. Clark*, N. 35. p. 681.

Then he sheweth, how the seminal vessels pass *è Testibus ad Epididymides*, vid. not by *one* Trunck (as Dr. *Highmore* thinks) but by 6. or 7. small *ductus*'s; assigning the cause, why Doctor *Highmore* did not see them.

Further he examines, An semen in testibus conficiatur; utrum ex Sanguine vel ex Lympha? quomodo elaboretur, crassescat, lactescat? qua via à Testibus ad Urethram excurrat.

Moreover he endeavours to prove, *Vesiculas seminales ordinatas esse non seminis generationi, sed receptioni & asservationi.*

He also observeth concerning the *seminal matter*, that 'tis composed *ex duplici materia*, which after *Aristotle*, he calls λόγον σπερματικὸν καὶ ὄγκον σπερματικόν, considering this twofold matter like *Dough* and *Ferment*, this infecting and quickning that, and the grosser part being a conservatory and vehicle to that, which is most elaborate.

When he examins the *Penis*, he taketh notice, *Omnes hactenus*

nus Anatomicos perperam aßignasse usum musculorum Penis, quos Erectores appellant; Eorum quippe provinciam non esse, Penem erigere, & dilatare Urethram, cum omnis Musculi actio sit contractio, quæ extensioni contraria est; eos potius Penem *versus interiora retrahere quam erigere: Interim, hosce Penis Musculos, coarctando corpora nervosa circa eorum exortum, materiam seminalem versus Penis partem anteriorem propellere, atque hac ratione corporum nervosorum distensione erectionem augere.*

Before we conclude this Account, we cannot but take notice, that the Author occasionally inserts in this Book divers curious and remarkable Examples and Observations; some whereof are.

1. Concerning those, that are born, either *absque Testibus*; or, *cum Testiculo uno*; or, *cum tribus, idque hæreditario per aliquot familias, admodum fæcundas.*

2. About the *situs præternaturalis Testiculorum, generationis tamen virtutem non impedientis.*

3. Concerning *lactescent Bloud* in a man living at *Delft* in *Holland*, whose Bloud alwayes turn'd into Milk, when let out either by venæ-sections, or by bleeding at the Nose, or by a wound. V. pag. 84, 85. Compare *Numb. 6.* pag. 100, 117, 118. and *Numb.* 8. pag. 139. of these *Transactions.*

4. Concerning the strange alteration made in Femals, *ab Aura seminali:* quod confirmat exemplo felis, diu sugentis (idque ad integram fere sui nutritionem) lac mammarum caniculæ, per aliquot annos à coitu prohibitæ; deinceps vero, postquam catella admiserat canem, nunquam ab eo tempore lac ex mammis ejus exsugere volentis.

5. About a strange *Hæmorrhagy* per *Penem*, which amounted to 14. pound, in a Porter of 52. years old, falling down with a heavy load upon a board, laid over a ditch, and so turning about, when the said porter trod upon it, that it cast him down upon its edge, turn'd between his legs; yet the Patient by the skill and care of our Author recover'd.

6. Various Observations of Clysters and Suppositories, cast up by Vomits, p. 195, 196.

7. Several wayes of performing unbloudy dissections of Animals, p. 228, 229, &c.

II. LO-

II. *LOGARITHMOTECHNIA NICOLAI MERCATORIS. Concerning which we ſhall here deliver the account of the Judicious Dr.* I. Wallis, *given in a Letter to the Lord Viſ-count* Brouncker, *as follows;*

Incidebam heri (Illuſtriſſime Domine) in D. *Mercatoris Logarithmotechniam*, nuper editam. Quæ ita mihi placuit, ut non prius dimiſerim quàm perlegiſſem totam. Et quamquam pauca quædam, Phraſeologiam quod ſpectat ſeu loquendi formulas nonnullas, mutata mallem; ſunt tamen ipſa ſenſu ſuo ſana: Eaque quæ ſuperſtruitur Doctrina, Logarithmos expedite atque ſubtiliter conſtruendi, perſpicue ſatis atque ingenioſe traditur.

Quæ huic ſubjungitur *Quadratura Hyperbolæ*, elegans admodum eſt atque ingenioſa. Nempe ad hunc ſenſum. V.Fig.1.

Poſtquam in Hyperbola MBF, (cujus Aſymptotæ AN, AH, ad angulum rectum coeunt) oſtenderat, prop. 14, Rectangula BIA, FHA, spA, &c. (ductis BI, FH, sp, &c, parallelis Aſymptotæ AN,) invicem eſſe æqualia; adeoque latera habere reciproce proportionalia; (quæ nota eſt Hyperbolæ proprietas:) Poſitis AI $=$ BI $=1$, & HI $=a$: oſtendit, prop. 15. FH $=\frac{1}{1+a}$ (Nempe propter analogiam AH. AI :: BI. FH. hoc eſt, $1+a.\ 1 :: 1.\ \frac{1}{1+a.}$ Sed & (quod Dividendo 1, per $1+a$ oſtenditur,) $\frac{1}{1+a}=1-a+a^2-a^3+a^4$ &c.] (continuatis deinceps, ipſius *a* poteſtatibus, alternatim negatis & affirmatis.)

$1+a)\,1\,(1,-aa^2,+,a^3,+,a^4,$ &c

$1+a$

$-a$

$-a\ -a^2$

$+a^2$

$+a^2\ +a^3$

$-a^3$

$-a^3\ a^4$

$+a^4$

&c.

Cumque hoc perinde obtineat, ubicunque ultra punctum I, ponatur H. Poſitis, ut prius AI $=1$; hujuſque continuatione qualibet, ut Ir $=$ A; quæ intelligatur in æquales partes innumeras dividi, quarum quælibet, ut Ip, pq, &c. dicatur *a*; adeoque Ip, Iq, &c, ſint *a*, 2a, 3a, &c. uſque ad A: Quæ his reſpondent rectæ ps, qt, &c. uſque ad *ru*, (ſpatium BI ru complentes) ſunt,

$$1 - a + a^2 - a^3 + a^4 \ \&c.$$
$$1 - 2a + 4a^2 - 8a^3 + 16a^4 \ \&c.$$
$$1 - 3a + 9a^2 - 27a^3 + 81a^4 \ \&c.$$

& ſic deinceps uſque ad

$$1 - A + A^2 - A^3 + A^4 \ \&c.$$

Cum itaque ſint

$$1 + 1 + 1 \ \&c. \text{(uſque ad ultimum)} = A$$
$$a + 2a + 3a \ \&c. \text{(uſque ad A)} = \tfrac{1}{2}A^2$$
$$a^2 + 4a^2 + 9a^2 \ \&c. \text{(uſque ad } A^2) = \tfrac{1}{3}A^3$$
$$a^3 + 8a^3 + 27a^3, \ \&c. \text{(uſqne ad } A^3) = \tfrac{1}{4}A^4$$

& ſic deinceps:

(quod oſtendit ille prop. 16, eſtque à me alibi demonſtratum:) Recte colligit, prop. 17. Expoſitum ſpatium Hyperbolicum BIru $= A - \frac{1}{2}A^2 + \frac{1}{3}A^3 - \frac{1}{4}A^4 + \frac{1}{5}A^5$, &c. Adeoque ſi (aſſignato, ipſi $A = I r$, valore ſuo in numeris, ut res poſtulaverit,) diſtribuantur in duas claſſes $A, \frac{1}{3}A^3, \frac{1}{5}A^5$, &c. (poteſtates affirmatæ,) & $\frac{1}{2}A^2, \frac{1}{4}A^4$, &c. (poteſtates negatæ;) harumque Aggregatum, ex Aggregato illarum, ſubducatur; Reſiduum erit ipſum BIru ſpatium Hyperbolicum.

Nequis autem operam luſum iri exiſtimet, propter Addendorum ſeriem in utraque claſſe infinitam; adeoque non abſolvendam: Hinc incommodo medelam (tacitus) adhibet: ponendo $A = 0.1$, vel $A = 0,21$, aliive fractioni decimali æqualem, adeoque minorem quam 1: (Hoc eſt, ſumpta Ir minore quam $AI = 1$.) Quo fit, ut poſteriores ipſius A poteſtates tot gradibus infra Integrorum ſedem deſcendant, ut merito negligi poſſint.

Exempli gratia; poſitis $AI = 1$, & $Ir = 0\,21$. erit

$A = 0,21$

$\frac{1}{3}A^3 = 0,003087$	$\frac{1}{2}A^2 = 0,02205$
$\frac{1}{5}A^5 = 0,000081682$	$\frac{1}{4}A^4 = 0,000486202$
$\frac{1}{7}A^7 = 0,000002572$	$\frac{1}{6}A^6 = 0,000014294$
$\frac{1}{9}A^9 = 0,000000088$	$\frac{1}{8}A^8 = 0,000000472$
$\frac{1}{11}A^{11} = 0,000000003$	$\frac{1}{10}A^{10} = 0,000000016$

$+ 0,213171345 - 0,022550984 = 0,190620361 =$ BIru

Quæ eſt brevis Synopſis Quadraturæ ſuæ ſatis elegans.

Diſſimulandum interim non eſt; ſiquis totius BIHF ſpatii (cujus latus IH longius intelligatur quam AI) quadraturam poſtulet; rem non ita feliciter ſucceſſuram: propter medelam, quam modo diximus, malo minus ſufficientem. Cum enim jam ponenda ſit $A > 1$; manifeſtum eſt, poſteriores ipſius poteſtates, altius in Integrorum ſedes penetraturas, adeoque minime negligendas.

Huic autem incommodo, levi conſtructionis immutatione, facile ſubvenitur. Vid. Fig. 1.

Cæteris utique ut prius conſtructis; Quadrandum exponatur HFur ſpatium

tium; (cujuscunque fuerit longitudinis A H; puta major minorve quam A I, vel huic æqualis: sumptoque ubivis inter A & H, puncto r; puta ultra citrave punctum I, vel in ipso I puncto:) Ponantur autem (non, ut prius $AI = 1$, & $Ir = A$: sed) $AH = 1$; & $Hr = A$, quæ intelligatur in æquales partes innumeras dividi, quarum quælibet sit a. Erunt itaque, post $AH = 1$, reliquæ deinceps decrescentes $1-a$, $1-2a$, $1-3a$, &c. usque ad $Ar = 1 - A$. Item, propter æqualia Rectangula FHA, urA, BIA, &c. puta, $= b^2$: Erit $HF = \frac{b^2}{1}$; reliquæque deinceps $\frac{b^2}{1-a}$, $\frac{b^2}{1-2a}$, $\frac{b^2}{1-3a}$, &c, usque ad $ru = \frac{b^2}{1-A}$ spatium HFur complentes. (Quæ omnia ostensa sunt, in mea *Arithmetica Infinitorum*, prop. 88, 94, 95.)

Factaque Divisione; reperietur $\frac{b^2}{1-a} = b^2 + b^2a + b^2a^2 + b^2a^3 + b^2a^4$, &c. Hoc est, b^2 in $1 + a + a^3 + a^2 + a^4$, &c. (sumptis ipsius a potestatibus continue sequentibus affirmatis omnibus.) Cumque de reliquis idem sit judicium; erunt rectæ omnes, ipsis HF & ru interjectæ,

$$
\begin{array}{l}
1-a)\,b^2\,(b^2, + b^2a, + b^2a^2, + b^2a \text{ \&c.}\\
\underline{b^2 - b^2a}\\
\quad + b^2a\\
\quad \underline{+ b^2a - b^2a^2}\\
\qquad + b^2a^2\\
\qquad \underline{+ b^2a^2 - b^2a^3}\\
\qquad\quad + b^2a^3\\
\qquad\quad \underline{+ b^2a^3 - b^2a^4}\\
\qquad\qquad + b^2a^4\\
\qquad\qquad \text{\&c.}
\end{array}
$$

$$
\left.
\begin{array}{l}
1 + a + a^2 + a^3 + a^4 \text{ \&c.}\\
1 + 2a + 4a^2 + 8a^3 + 16a^4 \text{ \&c.}\\
1 + 3a + 9a^2 + 27a^3 + 81a^4 \text{ \&c.}\\
\text{\& sic deinceps usque ad}\\
1 + A + A^2 + A^3 + A^4 \text{ \&c.}
\end{array}
\right\}\ \text{in } b^2.
$$

Omniumq; Aggregatū, $A + \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ &c, in $b^2 = FHru$. (per *Arithm. Infin.* prop. 64.)

Exempli gratia.
Positis $AH = 1$.
$Hr = A = 0,21$
$AI = b = 0,\ 1$
Adeoque $b^2 = 0,01$

Erunt $A = 0,21$
$\frac{1}{2}A^2 = 0,02205$
$\frac{1}{3}A^3 = 0,003087$
$\frac{1}{4}A^4 = 0,00048623 -$
$\frac{1}{5}A^5 = 0,000081682 +$
$\frac{1}{6}A^6 = 0,000014294 +$
$\frac{1}{7}A^7 = 0,000002573 -$
$\frac{1}{8}A^8 = 0,000000473 -$
$\frac{1}{9}A^9 = 0,000000088 +$
$\frac{1}{10}A^{10} = 0,0000000017 -$
$\frac{1}{11}A^{11} = 0,000000003 +$

Horum summa — 0,235722333
Ducta in $b^2 = 0,01$
Exhibet ——— 0,00235722333 = FHru

Qua-

Qualium 1. = A N G N { Quadrato, / Rhombo, } si angulus A sit { Rectus. / Obliquus.

Quæ quidem tam absoluta est tamque expedita Hyperbolæ quadratura, ut nesciam an melior sperari debeat.

Atque hæc sunt quæ hac de re scribenda duxi. Quæ si D. *Mercatori* impertiveris; non displicebit, credo, hæc suæ Quadraturæ facta accessio.

Posse hæc ad *Logarithmorum* inventionem accommodari, non est quod moneam: Sed & ad *Summam Logarithmorum* inveniendam: (quam inquirit ille prop. 19.) Nempe, Positis A H = 1, A I = I B = b, (ut prius) planoque B I H F = pl. Erit $pl - b^2 + b^3$ = B I p s + B I q t + B I r u, &c. usque ad B I H F. Si autem non ab ipsa B I incipiatur; sed ultra citrave, puta à *p s*: Posita p H = a & p s F H = p l. erit (universaliter) p s t q + p s u r &c (usque ad p s F H) = $pl - ab^2$: qualium 1, æquetur cubo ipsius A H.) Quod alias, si opus erit, demonstrabitur. Tu interim, Illustrissime Domine, Vale.

Oxon. d. 8. *Julii*, 1668.

Fig 1

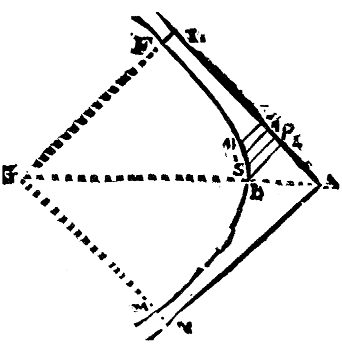
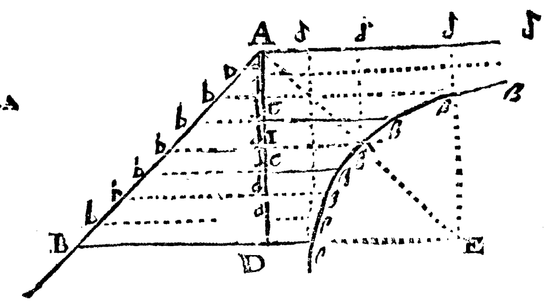


Fig 2



The Demonstration

Promised at the end of the foregoing Letter, follows in another from the same Author to the same Noble Lord, thus;

Petis (Illustrissime Domine) per literas tuas *Aug*.3. datas (quas hesterna nocte accepi) ut demonstrare velim methodum meam, Logarithmorum summam inveniendi, quam literis meis *Julii* 8. datis, brevissime insinuaveram.

Quæ quidem, cum sit cum Ungularum doctrina (quam alibi trado) connexa; opus erit ut utramque simul exponam: sed & rem totam (quam D. *Mercatoris* *natoris* figuræ & methodo quantum res ferebat accommodaveram) ad principia mea revocatam ab origine repetam. V. Fig. 2.

Ostensum est, in mea *Arithmetica Infinitorum*, prop. 95. Spatium Hyperbolicum ADβδδ (in infinitum continuatum à parte βδ, sed à parte Dβ ubivis terminatum,) Figuram esse quam ex *Primanorum Reciprocis* conflatam appello, Prop. 88. definitam: Cujus nempe Ordinatim—applicatæ dβ, dβ, sint Primanis (seu Arithmetice proportionalibus) db, db, (Triangulum complentibus) adeoque ipsis dA, dA, (suis à vertice distantiis) Reciproce Proportionales. Hoc est, (posito AD = d; & rectangulo ADβ = b^2; particulisque infinite exiguis a, a, &c;) si à vertice Aδ incipias $\frac{b^2}{o}$, $\frac{b^2}{a}$, $\frac{b^2}{2a}$, $\frac{b^2}{3a}$, &c. usque ad $\frac{b^2}{d}$ = Dβ: vel, si à base Dβ incipias, $\frac{b^2}{d}$, $\frac{b^2}{d-a}$, $\frac{b^2}{d-2a}$, $\frac{b^2}{d-3a}$, &c. usque ad $\frac{b^2}{d-d}$ = Aδ infinitæ, (nempe, si ad Verticem usque processum continuaveris;) vel, usque ad $\frac{b^2}{d-A}$ = Cβ, (posito DC = A,) si continuaveris usque ad Cβ, ubivis intra Aδ & Dβ sumptam. (Adeoque omnium Aggregatum, $\frac{b^2}{d} + \frac{b^2}{d-a} + \frac{b^2}{d-2a} + \frac{b^2}{d-3a}$, &c, est ipsum DCββ planum.)

Manifestum itaque est, (& ibidem prop. 94. ostensum) si intelligantur singulæ dβ, in suas à vertice distantias *Ad*, ductæ; hoc est, $\frac{B^2}{a}$ in *a*, $\frac{B^2}{2a}$ in 2a, (& sic de reliquis;) erunt omnia rectangula A dβ; hoc est, rectarum dβ momenta respectu Aδ, (intellige, facta ex magnitudine in distantiam ductâ;) seu plana semiquadrantalem Ungulam (cujus acies Aδ) complentia, (eisdem dβ rectis perpendiculariter insistentia;) invicem æqualia. Quippe singula = b^2. (Quorum cum unum sit AIVδ quadratum, erit AI = b.)

Adeoque Totius ADββδ (plani infiniti) seu omnium dβ illud complentium, momentum respectu rectæ Aδ, (ut axis æquilibrii;) seu Ungula semiquadrantalis eidem ADββδ insistens (aciem habens Aδ;) sunt totidem b^2; hoc est, d b^2. (Ungula magnitudinis finitæ plano infinitæ magnitudinis insistens.) Ejusque pars plano ACββδ insistens (propter AC = d — A.) similiter ostendetur æqualis ipsi d — A in b^2. ductæ; hoc est, d b^2 — A b^2. Adeoque pars reliqua, ipsi DCββ insistens, æqualis ipsi Ab^2. Quod itaque est ejusdem DCββ momentum respectu Aδ.

Atque hoc momentum per plani D C β β magnitudinem, puta per *pl*, divisum; exhibet plani distantiam Centri gravitatis ab A δ, $\frac{ab^2}{pl}$: adeoque distantiam ejusdem a D β, $d - \frac{ab^2}{pl}$.

Hæc itaque à D β distantia, in *pl* (plani magnitudinem) ducta; exhibet $dpl - Ab^2$ ejusdem D C β β momentum respectu D β; seu Ungulam eidem D C β β insistentem, cujus acies sit D β.

Hæc denique Ungula (cujus altitudo, in D β, nulla sit, sed, in C β, ipsi D C æqualis:) si ex planis ipsi D C β β parallelis conflari intelligatur; erunt ea, C D β β, C d β β, & sic deinceps; hoc est, aggregatum omnium C d β β, C d β β, usque ad C D β β.

Sunt autem ea plura (ut ex *Gregorii de Sancto Vincentio*, aliorumque post illum, doctrina constat) tanquam Logarithmi Arithmeticæ proportionalium C d, C d, &c. usque ad C D; (seu *a*, 2a, 3a, &c. usque ad A. Ergo Ungula ipsa, est eorundem Aggregatum. Hoc est (posito D = 1,) $dpl - Ab^2 = pl - Ab^2$. Quod ostendendum erat.

Porro; cum sit $\frac{b^2}{d-a}$ $(= d\beta) = \frac{b^2}{d} + \frac{ab^2}{d2} + \frac{a^2b^2}{d3} + \frac{a^3b^2}{d4}$ &c (Quod dividendo b^2 per $d-a$, patebit:) vel, posito $d = 1$, (quó ipsius *d* potestates omnes deleantur,) $b^2 + ab^2 + a^2b^2 + a^3b^2$ &c. seu $1 + a + a^2 + a^3$, &c. in b^2. & similiter $\frac{b^2}{d-2a} = \frac{b^2}{d} + \frac{2a\,b^2}{d^2} + \frac{4a^2\,b^2}{d^3} + \frac{8a^3\,b^2}{d^4}$ &c. $= b^2 + 2ab^2 + 4a^2b^2 + 8a^3b^2$ &c. $= b^2$ in $1 + 2a + 4a^2 + 8a^3$, &c. & similiter in reliquis:

Erunt omnes d β, (spatium D C β β complectentes,)

$$\left.\begin{matrix} 1 + a + a^2 + a^3 + a^4 \text{ \&c.} \\ 1 + 2a + 4a^2 + 8a^3 + 16a^4 \text{ \&c.} \\ 1 + 3a + 9a^2 + 27a^3 + 81a^4 \text{ \&c.} \\ \text{\& sic deinceps usque ad} \\ 1 + A + A^2 + A^3 + A^4 \text{ \&c.} \end{matrix}\right\} \text{in } b^2.$$

Adeoq; (per Arithm. Infin. prop. 64.) omnium Aggregatum, seu ipsum D C β β spatium, erit

$$A + \tfrac{1}{2}A2 + \tfrac{1}{3}A3 + \tfrac{1}{4}A4 + \tfrac{1}{5}A5 \text{ \&c, in } b^2 = pl.$$

Qualium 1 = A B E δ Quadrato vel Rhombo

Ideoque, Plani D C β β momentum respectu D β; seu semiquadrantalis Ungula eidem insistens cujus acies sit D β; seu planorum aggregatum ipsam constituentium; seu Logarithmorum summa quos ea representant, $dpl - Ab^2 = pl - Ab^2, = \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ in b^2:

Qualium

Qualium Cubus (seu Rhombus solidus) A D E ♪ sit 1.

Si vero non ponatur d = 1, sed cujuscunque magnitudinis: erit saltem

$$\frac{A}{d} + \frac{A^2}{2d^2} + \frac{A^3}{3d^3} + \frac{A^4}{4d^4} \text{ \&c. in } b^2. = p\,l.$$

Vel (posito $\frac{A}{d} = e$) erit $e + \frac{1}{2}e^2 + \frac{1}{3}e^3 + \frac{1}{4}e^4$ &c. in $b^2 = pl$. Qualium d^2 = A D E ♪ Quadrato vel Rhombo.

Ungulaque (ut prius) d p l — A b^2. Qualium d^3 = A D E ♪ Cubo, vel (si angulus A sit obliquus) Rhombo solido.

Cumque A (posito d = 1) vel *e* (quicunque ponatur valor ipsius *d*) sit minor quam 1, (propter A < d:) illius potestates posteriores ita continue decrescunt, ut tandem negligi possint; planique valor *pl.* exhibeatur quantumlibet vero propinquus.

Atque hæc est, Illustrissime Domine, Methodi, quam innuebam, ex meis principiis deductio, & demonstratio brevis. Vale. *Oxon.* d. 5. *Aug.* 1668.

Some Illustration Of the Logarithmotechnia *of M.* Mercator, *who communicated it to the Publisher, as follows.*

Si quorum in manus incidit Logarithmotechnia mea, non inviti, opinor, adspicient paucula hæc exempla, miram istius methodi facilitatem cum summa præcisione conjunctam ostendentia.

Expo-nentes	Unitatis ordo		Binarii ordo	
1	1	1	2	2
2	1	0, 5	4	2
3	1	0, 333333	8	2, 666666
4	1	0, 25	16	4
5	1	0, 2	32	6, 4
6	1	0, 166666	64	10, 666666
7	1	0, 142857	128	18, 285714
8	1	0, 125	256	32
9	1	0, 111111	512	56, 888888
10	1	0, 1	1024	102, 4

Duo sunt ordines tabellæ, prior unitatis, alter binarii propago, quorum uterque denorum numerorum primorum Log-os producit, præter compositorum Log-os, qui & ipsi requiruntur.

Ex

Ex primo ordine

1
05
033333333
025
02
016666
01428
0125
011
01

+ 10033534772
— 502516792

$10536051564\frac{9}{10}$
$9531017980\frac{10}{11}$

1
.05
.0333333
025
02

+ 10000333353
— 500025

$10050335853\frac{99}{100}$
$9950330853\frac{100}{101}$

Parimodo ex eodem ordine procedunt rationes $\frac{999}{1000}$, $\frac{1000}{1001}$, $\frac{9999}{10000}$, $\frac{10000}{10001}$, $\frac{99999}{100000}$, $\frac{100000}{100001}$.

Ex fecundo ordine.

2
2
266666666
4
64
1066666
182857
32
5689
1024
186
341
630

+ 20273255404
— 2041099724

2
.2
.2666666
4
.64
10

+ 20002667306
— 200040010

$20202707316\frac{98}{100}$
$19802627296\frac{100}{102}$

Haud fecus ex eodem ordine eliciuntur rationes $\frac{998}{1000}$, $\frac{1000}{1002}$, $\frac{9998}{10000}$, $\frac{10000}{10002}$, $\frac{99998}{100000}$, $\frac{100000}{100002}$, &c.

1		$22314355128\frac{8}{10}$
2		$18232155680\frac{10}{12}$
3	1 + 2	$40546510808\frac{8}{12} = \frac{2}{3}$
4	ex pc. pag.	$10536051564\frac{9}{10}$
5	2 + 4	$28768207244\frac{9}{12} = \frac{3}{4}$
6	3 + 5	$69314718052\frac{2}{4} = \frac{1}{2} =$ L 2rii
7	6 × 3	$207944154156\frac{1}{8} =$ L 8rii
8	1 + 7	$230258509284\frac{1}{10} =$ L 10rii
9	ex pc. pag.	$9531017980\frac{10}{11}$
10	8 + 9	$239789527264\frac{1}{11} =$ L 11rii
11	3 + 6	$109861228860\frac{1}{2} + \frac{2}{3} =$ L 3tii

Simi-

Similes ordines à 3rio, 4rio, & quovis alio numero derivari poſſunt, ſuas quiſque rationes exhibiturus.

Acquiſito Log-o 10rii, conficienda eſt ſtatim tabella reducendorum Log-orum naturalium ad Tabulares, ut quævis ratio, ſimul ac inventa eſt, reducatur ad menſuram tabularium; ita enim Log-i compoſitorum, quorum ope ad primorum Log-os deſcenditur, ſimul fient Tabulares abſque reductione.

Fiat igitur, ut Log-us 10rii non-tabularis 2302585, ad tabularem 10000000; ita 1, ad 4,3429448. Hic numerus bis, ter, quater & pluries ſumptus conſtituit tabellam reducendorum Log-orum naturalium ad tabulares, quam hic ſubjectam vides.

1	043429448190
2	086858896380
3	130288344570
4	173717792761
5	217147240951
6	260576689141
7	304006137332
8	347435585522
9	390865033712

Hujus igitur ope tabellæ, rationis $\frac{98}{100}$ menſura naturalis 20202707316 reducitur ad tabularem hoc modo:

2	086858896381
0	0
2	0868588964
0	0
2	08685890
7	3040061
0	0
7	30401
3	1303
1	043
6	26
	87739243069

Tum à Log-o 100rii 20000000000000
auferatur rationis $\frac{98}{100}$ menſura 87739243069
reſtat 19912260756031 = L 98
unde ablato Log-2ii 3010299956640
reſtat ——— 16901960800291 = L 99
cujus ſemis ——— 8450980400145 = L 7

Item rationis $\frac{100}{102}$ menſura naturalis 19802627296 reducta, fit 86001717619.

Ergo jà Log-o 100rii 20000000000000
adde rationis $\frac{100}{102}$ menſura 86001717619
fit ——— 20086001717619 = L 102
unde ablato Log-o 6rii 7781512503836
reſtat ——— 12304489213783 = L 17

Hic tabula numerorum primorum egregium uſum præſtare poteſt.

Sed & ejuſdem primi 17 Log-um abſque ambage invenire datur, dicendo: 20. 17 :: 10. 8|5; tum differentiæ inter 10 & 8|5 (nimirum 1|5) ſumendo quadrati ſemiſſem, cubi trientem, &c. tractandoque iſtum ordinem, ut ſuprà, inveniemus ſimul Log-os abſolutorum 23, 197, 203, 1997, 2003, &c.

1	1, 5	1, 5
2	2, 25	1, 125
3	3, 375	1, 125
4	5, 0625	1, 265625
5	7, 59375	1, 51875
6	11, 390625	1, 8984375

15
1125
1125
1265625
1518
189
22

+15114040
— 1137845

16251885 $\frac{17}{20}$
13976195 $\frac{20}{23}$

Cæterùm isthæc omnia, & longè plura ex prop. 13, 15, & 16 Logarithmotechniæ nostræ apertè derivantur, non magis considerando hyperbolam, quàm si ea nusquam in rerum natura extitisset. Quare frustra sunt, qui hyperbolam ad constructionem Logarithmorum vel hilum conferre autumant; imo Logarithmorum ope quadrare hyperbolam, verius est. Id quod exemplo ostendere haud pigebit. In diagrammate (Fig. 1.) sit AH 74305816 partium, qualium AI est 1, & oporteat invenire aream BIHF.

Opus est ad eam rem tabella subjecta, quæ continet Log-os naturales suprà acquisitos, in priori columna ab 1 usque ad 9, in altera à 10 usque ad 1000000000.

1	00000000000	02, 30258509299
2	69314718052	04, 60517018599
3	109861228860	06, 90775527898
4	138629436104	09, 21034037198
5	160943791232	11, 51292546497
6	179175946912	13, 81551055796
7	194591014904	16, 11809565096
8	207944154156	18, 42068074395
9	219722457720	20, 72326583695

Tum prima figura numeri dati semper distinguatur à sequentibus separatrice, hoc modo: 7, 4305816, & ipsi primæ figuræ semper adjiciatur 1, ita conflantur, hoc loco, 8. Quærenda est nunc rationis 8 ad 7,4305816 mensura naturalis. Id ut fiat commodius, dic: ut 8 ad 7, 4305816; ita 1 ad 0, 9288227, hunc quartum proportionalem aufer ab 1, reliquum 0, 0711773 voco potestatem primam, quæ ducenda est in se ita, ut in facto idem numerus partium extet, qui erat in ipso 0, 0711773; productum 0, 0050662 est potestas secunda, quæ rursus ducatur in primam 0, 0711773, ut idem numerus partium exstet, prodit 0, 0003606, quæ est tertia potestas; & eodem modo invenitur quarta 0, 0000256, & quinta 0,0000018. Deinde

Potestas

Potestas prima	0, 0711773	addantur
Et secundæ semis	25331	
Et tertiæ triens	1202	
Et quartæ quadrans	64	
Et quintæ pars quinta	4	

summa —————————0,0738374 est mensura rationis 8 ad 7,43058 16, eadem scilicet cum ratione 80000000 ad 74305816. Porro Log-us absoluti 80000000 facile acquiritur ex superiori tabella; cum enim index primæ figuræ numeri 80000000 sit 7, è regione 7rii ex secunda columna excerpo Log-um absoluti 10000000 (hoc est unitatis septem cyphris affectæ)

qui reperitur	16, 11809565	addo
cui subscribo Log-um 8vii	2, 07944154	
summa est Log-us absoluti 80000000	= 18, 19753719	
ablata mensura rationis 80000000 ad 74300581 6	= 0, 0738374	
restat Log-us absoluti 74305816	= 18, 1236997,	atque

tanta est area BIHF.

Mantissæ loco accipe modum facillimum quadrandi quamvis hyperbolæ partem per Log-os tabulares. Dati numeri 74305816 Log-us tabularis est 7,87102278, per superioris tabellæ columnam secundam reducendus ad naturalem, proditque eadem, quæ supra, area BIHF = 18, 123699872.

Postremo, ne quis hæsitationi locus restet, accipe, quo pacto ex Prop. 13, 15, 16. Logarithmot. calculum superiorem derivem.

Differentia terminorum rationem quamvis exprimentium si concipiatur divisa in partes æquales innumeras; composita erit ratio tota extremorum terminorum ex innumeris ratiunculis terminorum à minimo ad maximum infinitissima parte ipsius differentiæ se mutuo excedentium. Sin iidem illi termini innumeri accipiantur pro mediis Arithmeticis aliorum terminorum simili parte infinitissima distantium; summa omnium ratiuncularum posterioribus hisce terminis intercedentium deficiet à tota ratione extremorum, non nisi semisse primæ & ultimæ ratiuncularum à prioribus terminis contentarum, id est, ratiuncula minori, quam quæ ullis numeris exprimi possit. Quare posito Maximo termino = 1, & parte infinitissima differentiæ = i, & mensura rationis minimæ itidem i; erit ut medium Arithmeticum terminorum rationis minimam proxime præcedentis, ad medium Arithmeticum terminorum ipsius minimæ; ita mensura minimæ, ad mensuram proxime majoris, hoc est:

$1 - i \,.\, 1 :: i \,.\, i + ii + i^3 + i^4$ &c. mensuræ ultimæ
$1 - 2i \,.\, 1 :: i \,.\, i + 2ii + 4i^3 + 8i^4$ &c. penultimæ — add.
$1 - 3i \,.\, 1 :: i \,.\, i + 3ii + 9i^3 + 27i^4$ &c. antepenultimæ

sit summa ratiuncul. $= 3i + 6ii + 14i^3 + 36i^4$ &c. = numero terminorum,

rum, plus ſumma eorundem terminorum, plus ſumma quadratorum ab iiſdem, &c.

Sin minimus terminus ponatur $=$ 1, manentibus cæteris ut ſupra; evadit ſumma ratiuncularum $= 3i - 6ii + 14i^3 - 36i^4$, &c.

Hinc data differentia terminorum $= 0|\underline{1}$, erit numerus terminorum $= 0|\underline{1}$, & per 16 Logarithmot. ſumma eorundem terminorum $= 0,005$, & ſumma quadratorum $= 0,000333$. At data differentia terminorum $= 0|\underline{10}$; numerus terminorum eſt $= 0,01$, & ſumma eorundem $= 0,00005$, & ſumma quadratorum $= 0,00000333$, &c.

Nota. Prop. IV. Logarithmot. Signa ſpeciebus intercedentia debebant eſſe alternatim affirmata & negata: atque ubicunque, Lector offenderit *infinitiſſimam*, legat *infiniteſimam*.

Errata.

Page 742. *l.* 25. put a *comma* after *open'd*, (which is material for the ſenſe.) p. 749. *l.* 16. r. *idque*. ibid. *l* 40. r. *magnitudinum*. p. 753. *l.* 20. r. $- a + a^2, - a^3$, p. 754. *l.* 19. r. *Huic*. p. 755. *l.* 11. r. $b^2 a^2 + b^2 a^3 + b^2 a^4$. ibid. *l.* 14. r $+ a^2 + a^3$. p. 756. in Fig. 1. the letters appearing obſcure, thoſe, that denote the ſmall lines parallel to the Aſymptote N A, are I B. *ps. qt. rn.* And the other capital letters are G F H. G B A. G M N.

In the *SAVOY*,

Printed by *T.N.* for *John Martyn*, Printer to the *Royal Society*, and are to be ſold at the *Bell* a little without *Temple-Bar*, 1668.